Structural Recovery (Physical Ageing) of the Friction Coefficient of Polymers

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Received 1 July 2007; accepted 2 April 2008
DOI: 10.1002/polb.21468
Published online in Wiley InterScience (www.interscience.wiley.com).

ABSTRACT: Most of the mechanical properties of the polymers are subject to structural recovery (physical ageing). The aim of this work was, therefore, to evaluate the sensitivity of the friction coefficient to physical ageing. The apparent friction coefficient is the ratio of the tangential force to the normal load applied to a moving tip in contact with the surface of a material. This coefficient includes a “true friction” at the interface and a “geometrical friction,” which is the flowing and bulk dissipative effect. In the case of solid polymers, the material underneath the moving tip may display various types of behavior: elastic, viscoelastic, elastoplastic (elastic and plastic strains are present in the contact area), or fully plastic. Scratching and sliding experiments were performed on PMMA to determine the true friction coefficient over a wide range of contact deformations. An analysis of the true friction coefficient as a function of the physical ageing (structural recovery) showed that the friction decreases with increasing structural recovery of the polymer in the case of sliding. Whatever the level of physical ageing, at a high contact strain (fully plastic contact), the true friction coefficient tended to a unique and high value independent of the thermal history of the polymer. © 2008 Wiley Periodicals, Inc. J Polym Sci Part B: Polym Phys 46: 1337–1347, 2008
Keywords: ageing; shear; yielding

INTRODUCTION

The influence of physical ageing (structural recovery) on the mechanical properties of the polymers has been widely studied for more than 30 years. However, no attempts appear to have been made to evaluate the influence of structural recovery on the friction properties of the materials. This influence may take its origin on three scales usually attributed to friction phenomena.

The statistical macroscopic scale is that of the relative motion of two large bodies with some roughness. At low pressure, the contact area is composed of a large number of elementary discontinuous local contact areas having various geometries. Measurements of the macroscopic friction reveal a dependency on temperature and sliding speed and attempts have been made to give a physical sense to this dependency. These attempts were generally based on previous work on the statistical roughness of a surface with inclusion of some single contact results. Because the elastic moduli of the materials are a major parameter governing the friction in multicontact models, physical ageing may influence friction through the variation of the elastic modulus during ageing.

On a microscopic scale, the single contact area (as opposed to a multicontact area) may be
considered to be a sufficiently smooth surface with perfectly continuous contact between the bodies to allow measurement of a local friction coefficient. On solid polymer surfaces this local friction displays a peak at the glass transition temperature, and the evolution of the friction is comparable to that of the mechanical loss factor $\tan \delta$ attributed to adhesion hysteresis.9–11 At this microscopic level, the dimensions of the microstructure of the polymer are still much smaller than the contact area and continuum mechanics may be applied. The local contact pressure and contact strain may therefore be considered as relevant parameters. At this level,12–14 microscratching or more recently nanoscratching are the most common experimental methods. Briscoe15 assumed that the energy consumed is mainly located in two zones. The first is the interface, a very thin layer subject to extremely high shear strain, a high strain rate, and adhesive slipping. The second zone is roughly spherical with a size much larger than the interface and comparable to that of the groove left on the surface. The influence of physical ageing on the local friction coefficient becomes quite complex on this scale. Young’s modulus, the yield stress, and even molecular orientations at the interface may be affected by the structural recovery.

On the molecular scale, using a spherical tip with a large radius and low normal loads (as in SFA friction tests16), sliding studies on rubber materials have shown that the friction depends on the sliding speed in relation to the interpenetration of the macromolecular chains, while the adhesion hysteresis is linked to the work dissipated when the chains remain in their original state.17 Friction tests performed under an AFM may be used to study the friction at the level of a few polymer chains or to extract a molecular chain. It has been observed that the friction is stable only after a sliding length approximately equal to the contact width.18 The AFM is also used to study nano and microscratches and to determine the surface topography of a sample,19 as are other micro and nanoscratch devices. However, the results obtained on this scale are sometimes unpredictable. The friction may be independent of the sliding speed,20 while the depth of the groove can vary with the scratching speed.21 On the molecular friction scale, it is difficult to adjust and control the contact pressure as one cannot easily change the radius of the mechanical probe. This limitation is acceptable for analysis of the friction of a lubricated film or rubber material, but it is unacceptable for a solid polymer where the contact pressure controls the strain and yielding.

Hence, the intermediate microscopic scale of a single contact would seem to be well suited to analyze the relationship between the true interfacial friction coefficient, the yielding and the ageing of surface properties. This article presents an analysis of the contact pressure and true friction coefficient determined from sliding and scratching experiments on PMMA surfaces and discusses the experimental relationship between physical ageing and friction. The measured values of the true friction coefficient as a function of the ageing and yielding during sliding and scratching will be presented. On the other hand, it is known that physically aged polymers can be rejuvenated by mechanical yielding1,22 Depending on the value of the true friction coefficient, yielding may appear during sliding either at the surface or in the subsurface of the contact area.23 In this work, purely elastic and elastic–plastic contacts and likewise surface and subsurface yielding were explored numerically using finite element simulations.24

**EXPERIMENTAL APPARATUS AND TEST CONDITIONS**

The scratch apparatus is based on a commercial servomechanism bearing a small transparent environmental chamber containing the sample and the moving tip. A built-in microscope allows in situ control and analysis of the groove left on the surface, which is possible due to the transparency of the polymers tested. Scratch tests may be performed over a wide range of tip speeds (1–15 mm/s) and within a temperature range covering the $\alpha$ and $\beta$ transitions of common polymers (−70 °C to +120 °C). Control of the moving tip and recording of the load, speed, and temperature are computer driven. The normal load applied to the tip can be selected from 0.05 N to 25 N. The test parameters are the normal load, tip geometry, temperature, and sliding speed, while the measured parameters are the tangential force, groove geometry, and true contact area. In these experiments, at each loading step and throughout the scratching process, in situ photographs were taken to save information on the shape of the true contact area. Figure 1 presents a schematic diagram of the apparatus and its geometrical parameters.
The material employed in this study was a commercial grade of extruded polymethylmethacrylate (PMMA) having a molecular weight of 10^5 g/mol. Its mechanical properties were determined by dynamic mechanical analysis (DMA) using an Instron tensile device. Figure 2 shows the Young's modulus as a function of temperature for various frequencies and Figure 3 the Young's modulus estimated at one frequency as a function of ageing. The ageing time of a polymer sample was defined as the time elapsed because cooling after the heating cycle imposed to rejuvenate the material. Throughout each test, the sample remained mounted in the experimental chamber and the temperature was kept constant at 30°C. The material displayed the usual structural recovery behavior: an increase in the storage modulus and a decrease in the loss modulus.

To analyze the evolution of the true fiction coefficient with the ageing of a polymer, the samples were prepared in a specific manner. No solvent or water was used during cleaning of the surfaces as they were gently sandpapered with a very fine sandpaper to remove a thin layer of polluted matter. The samples were then heated for 120 min to 50°C above their glass transition and squeezed between two glass plates to create a new rejuvenated surface. After cooling, the samples were stored (aged) at 30°C or 60°C under vacuum until scratching experiments. To obtain a contact and groove without wear, the moisture content of the storage atmosphere had to be less than 5%. Scratch tests were performed at 30°C, the speed of the moving tip was kept constant at 0.03 mm/s and the same experimental procedure was used for all samples. After starting the passage of the tip, a normal load was applied to it and increased stepwise from 0.05 N to 25 N. The tips were made of polished stainless steel, except the smallest one (radius of 116 μm) which was diamond. A large panel of tip radii was employed to cover a wide range of strain and pressure. The tip with a ra-

![Figure 1](image1.png)

**Figure 1.** Principle of the scratch test and diagram of the *in situ* observation device.\textsuperscript{14} The main geometrical parameters are defined on the schema.

![Figure 2](image2.png)

**Figure 2.** Elastic modulus of the PMMA polymer used in this study. The glass temperature is about 110°C.

*Journal of Polymer Science: Part B: Polymer Physics* DOI 10.1002/polb

![Figure 3](image3.png)

**Figure 3.** Evolution of Young's modulus and the loss factor with ageing of the polymer.
The contact area, while the tip with a radius of 400 µm generated plastic flow beneath the contact, and those having radii exceeding 400 µm, that is, 5 mm and 12.5 mm, generated elastic deformation in the volume under the contact area. The ball tips were polished and their roughness remained less than 20 nm. Before tests, the tips were cleaned with alcohol and dried under a nitrogen flow.

EFFECT OF PHYSICAL AGEING ON THE FRICTION BEHAVIOR

Case of a Surface with Very Long Structural Recovery

These experimental results were obtained with very “old” PMMA (some years old, with a poorly known thermal history). Figure 4 presents the interfacial shear stress plotted as a function of the mean contact pressure in the case of scratching with a fully yielded contact (high normal load applied to the moving tip). This type of master curve was obtained using a technique described in 11 and Appendix 1, over a large temperature range. Measurements of this kind allow one to calculate the true local friction coefficient as a function of temperature and pressure. It appears that at low temperature and high pressure, over a wide range of temperatures and pressures, the true local shear stress increases linearly with the contact pressure, which means that the true local friction coefficient is constant. It is roughly 0.5 below 80 °C for fully yielded PMMA. Above and close to the glass temperature the shear stress does not increase linearly with the contact pressure: at very low pressure and high temperature the true local friction coefficient is fairly high (>1 at 115 °C and 10 MPa) and decreases gradually to 0.5 at 80 °C and 150 MPa. In the pressure ranges which could be explored at several temperatures (close to 50 MPa and from 150 to 500 MPa), the true local friction coefficient does not seem to depend on temperature: for example, at 200 MPa the friction is the same at 20 °C and 80 °C, while at 50 MPa it has the same value at 80 °C (below Tg) as at 115 °C (above Tg).

These results will be compared later to those for newly rejuvenated and stepwise aged PMMA surfaces. Thus, it is known that yielding rejuvenates the surface of glassy polymers.1,22 As these samples had been aged for a long time and the friction was measured at high loading with a fully yielded contact, the friction coefficient may be expected to be similar to that of a newly created surface tested at the same pressure.

Case of a Newly Created Surface

In Figure 4 both the temperature and pressure are variable, giving an overall view of the friction mechanism. However, a detailed analysis of the ageing effect requires results at constant temperature and variable pressure. The pressure may be varied over a certain range by changing the applied load, but also by changing the radius of the moving stainless steel tip. Figure 5 shows the true shear stress at 30 °C and a

![Figure 4](image1.png)

**Figure 4.** True interfacial shear stress versus contact pressure. Scratch tests on PMMA were performed with a conical tip at temperatures in the range −60 °C to 115 °C. Whatever the strain rate or temperature, these contacts were fully yielded.

![Figure 5](image2.png)

**Figure 5.** Interfacial shear stress versus contact pressure for a thermally rejuvenated polymer sample.
tip speed of 30 μm/s as a function of the mean contact pressure, which was estimated from the load applied to the spherical tip and the geometrical information obtained from in situ photographs for a wide range of tip radii. These radii were selected to generate both elastic sliding and plastic scratching. At low pressure and for elastic sliding of the moving tip the friction is about 0.8, while it decreases progressively to a constant value of about 0.45 above 100 MPa and for fully yielded scratching. At high pressure the friction on a new surface thus appears at first glance to be comparable to that on an aged surface, the variation from 0.45 to 0.5 in the slope being attributable to experimental scattering. This is a preliminary indication that the surface is rejuvenated by the yielding occurring during the scratching process at high pressure.

Ageing for the Same Duration at Different Temperatures

The surfaces were cleaned using the same procedure as before and after the thermal cycle to rejuvenate the polymer, the samples were aged for 3 months at 30 or 60 °C under vacuum. After ageing, friction tests were performed at 30 °C and a sliding speed of 30 μm/s. Testing these surfaces at the same temperature after ageing was the best way to analyze the effect of the structural recovery on the true friction coefficient, without interference from the temperature dependence of the friction (as in friction tests at various temperatures). Figure 6 shows the interfacial shear stress as a function of the contact pressure.

There are several noticeable features on this plot:

- At very high pressure, when scratching and yielding occur, the friction behavior is the same for rejuvenated and aged surfaces.
- Three months ageing at 30 °C does not change the friction properties of a rejuvenated surface.
- At very low pressure, when sliding occurs and the contact is elastic, the friction (0.15) of a 3-month-aged surface is 5–6 times lower than that of a rejuvenated surface (0.8).

True Friction Coefficient as a Function of the Normalized Contact Pressure

The friction behavior appeared to be quite different depending on whether or not yielding had occurred at the interface between the moving tip and the surface. Therefore, uniaxial yield stress measurements were performed under compression at several temperatures and strain rates. The experimental device was based on the moving cross head of an Instron tensile machine and the whole apparatus was enclosed in an Instron environmental chamber. Cylindrical samples 12.5 mm long and 5 mm in diameter were employed and tests were arranged out between −20 and +120 °C at four strain rates between \(10^{-4}\) and \(10^{-1}\) s\(^{-1}\). This allowed estimation of the yield stress over a wide range of scratching speeds and temperatures. Figure 7 shows the yield stress estimated from these tests as a func-
tion of the strain rate. Using these values, an extrapolated function was derived which allows one to estimate the yield stress to a good approximation over a wide range of temperatures and scratching velocities. Thus, the yield stress was fitted with a second degree polynomial law to estimate the values at strain rates comparable to those in scratch tests:

\[
\sigma_y(\dot{\varepsilon}, T) = a(T) + b(T)\log \dot{\varepsilon} + C(T)(\log \dot{\varepsilon})^2
\]

(1)

The mean contact strain rate is defined as the ratio between the sliding speed and the length of the contact area and ranges from 0.015 to 0.5 s\(^{-1}\).

Figure 8 depicts the true friction coefficient as a function of the normalized contact pressure, which is the ratio of the contact pressure to the yield stress determined at the same strain rate and temperature.\(^{25}\) The friction coefficient \(\mu\) is the ratio of the interfacial shear stress \(\sigma_{\text{int}}\) to the pressure \(p\) and is generally written as the sum of an adhesive effect \(\sigma_{\text{adh}}\) and a friction effect \(x\):

\[
\sigma_{\text{int}} = \sigma_{\text{adh}} + xp \quad \text{and} \quad \mu = \sigma_{\text{int}}/p = x + \sigma_{\text{adh}}/p
\]

(2)

In the case of amorphous glassy polymers, the adhesive effect \(\sigma_{\text{adh}}\) is very weak and may be neglected. However, at very low pressure \(\sigma_{\text{adh}}/p\) diverges and introduces scatter into the measurements.

As noted earlier, 3 months ageing at 30 °C does not significantly affect the friction properties. Three domains may be distinguished:

- At a very low normalized contact pressure (below 0.1), the true local friction is highly scattered as the pressure is close to zero and the ratio of scission to pressure diverges. Hence points below a normalized pressure of 0.1 are not shown.
- At an intermediate normalized contact pressure (between 0.1 and 1), the high friction of a rejuvenated surface decreases continuously with pressure, whereas the low friction of an aged surface remains more or less constant while increasing slowly.
- At a high normalized contact pressure (above 1, fully yielded contact), the true friction coefficient tends to a unique value of about 0.45, whatever the ageing. As shown in Figure 4, this corresponds to the true friction coefficient estimated from scratching.\(^{26,27}\)

STRESS AND STRAIN FIELD

The results presented earlier demonstrate that the mean pressure close to the contact controls the friction coefficient. Therefore, the pressure field must be calculated more precisely, in particular for the situation where the yield point is reached at the interface between the surface and the moving tip. The pressure field has been modeled previously using the CAST3M code\(^ {24,25,28}\). The finite element mesh was a right-angled parallelepiped, the domain elements were three-dimensional meshes with 10-node tetrahedral and the mesh was refined under the contact area. Elliptical contact pressure and shear stress distributions were used to model the contact between a spherical tip and the surface. The flow stress was described by a simplified G’sell-Jonas law:\(^ {29}\)

\[
\sigma = k\dot{\varepsilon}_{\text{vp}}^m h_{\text{g}}^{1/2}\dot{\varepsilon}_{\text{vp}}^2 \quad \text{which implies} \quad \sigma_{\text{yield}} = k\dot{\varepsilon}_{\text{vp}}^m
\]

(3)

where \(\dot{\varepsilon}_{\text{vp}}\) and \(\dot{\varepsilon}_{\text{vp}}\) are, respectively, the generalized viscoplastic strain rate and strain and \(k\) is the consistency, \(h_{\text{g}}\) the strain hardening coefficient, and \(m\) the sensitivity to the strain rate. In our simulation, the elastic recovery was directly related to the ratio of the flow stress \(\sigma\) to Young’s modulus \(E\), thermal effects were neglected and \(a_T\) was equal to 0. The three parameters \(k\), \(h_{\text{g}}\), and \(m\) have been described previously for PMMA\(^ {30,31}\) and were determined by an inverse method adapted to large deformations and based on interpretation of the force-

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Figure 8. True friction coefficient versus normalized contact pressure for variable ageing of the polymer sample.
penetration curves in indentation tests with two indenter shapes: $m = 0.1$, $h_g = 0.5$, and $k = 160 \text{ MPa s}^{-m}$. Numerical simulations were performed to locate the boundaries between elastic and elastoplastic contact and between elastoplastic and plastic contact. The first boundary could be simply related to the first mesh having a strain higher than the yield strain, while the second was defined to occur when all the matter contained in the half spherical volume under the contact area flowed plastically.

The results of the finite element simulation are presented in Figure 9, superimposed on the experimental results. Our findings agree with those reported by Johnson.\textsuperscript{23} At low values of the friction coefficient yielding starts under the surface (and not at the surface), whereas for higher values of the friction coefficient yielding starts at the interface between the moving tip and the surface.

**DISCUSSION**

**Reproducibility and Pollution of the Surface**

As the friction properties of a surface are extremely sensitive to surface pollution, all experiments were carried out very carefully, keeping the samples in a clean and controlled environment. Nevertheless, during ageing low molecular weight species or other unknown components existing inside the material could migrate to the surface and modify the friction behavior. The evolution of the molecular weight of the surface chains with storage of the polymer samples was therefore analyzed by size exclusion chromatography. The surface of the aged polymer was scraped to obtain a few milligrams of material for chromatography. A bulk sample was also taken to check for any evolution of the bulk molecular weight. There was no difference between the surface and bulk samples and whatever the storage temperature the molecular mass was equal to $10^5 \text{ g/mol}$. One should note that the polydispersity index of this polymer is about 2. However, the best proof of the absence of surface pollution was obtained by measuring the friction properties of an aged sample following purely thermal rejuvenation. If small molecules had migrated to the surface and caused a decrease in the friction coefficient, their influence should not be erased by this purely thermal rejuvenation. As seen in Figure 10, results for a newly created surface and for the aged surface thermally rejuvenated were identical within the scatter range.

**Does Ageing Really Control Friction?**

The effect of physical ageing on mechanical properties is usually a continuous and approximately linear function of the logarithm of time (Fig. 3). Unfortunately, the variation of the friction coefficient does not seem to follow such a simple law with increasing ageing time: 3 months ageing (4–5 decades longer than the quenching time) at 30°C changes the tensile modulus by about 10% but has no visible effect

![Figure 9](image_url1)

*Figure 9.* Plasticity beneath the contact. The transitions between elastic, elastoplastic, and plastic contact were identified by FEM and representative pictures of the yielded volume are shown.

![Figure 10](image_url2)

*Figure 10.* Comparison of the friction properties of a newly created surface with those of the same surface aged and thermally rejuvenated. The friction behavior is not modified by the migration of small molecules to the surface.
on the friction coefficient, whereas 3 months ageing at 60 °C has a dramatic effect on the friction. Therefore, another type of experiment was carried out to confirm the influence of the structural state on the friction.

The glass transition temperature (measured by DMA) of this polymer is roughly 110 °C at $10^{-2}$ s$^{-1}$ and 100 °C at $10^{-7}$ s$^{-1}$ (at 3 months). All previous experiments shown in this work were performed on samples annealed far above 110 °C and quenched within a few minutes, with the result that their surface structure was frozen at about 110 °C. Another newly created surface was maintained for 3 months at 100 °C (where it has reached the glass transition and has a rubbery consistency) and then quenched as usual. The structure of this new surface was at equilibrium at 100 °C instead of 110 °C. Figure 11 compares the friction coefficients at 30 °C (without ageing) for polymer samples at equilibrium at 100 and 110 °C. The friction is much lower for the sample having a structure frozen at 100 °C, clearly demonstrating that the structural state at the molecular level has a strong influence on the friction behavior at low pressure.

Overall Mechanism of the Influence of Ageing on the Friction Coefficient

The friction coefficient of a polymer depends on the temperature, pressure and sliding velocity. As seen earlier, the way physical ageing affects friction would appear to be quite complex, more so than the effect of ageing on DMA measurements. The basic reason is that DMA measurements are made at very low stress and strain, whereas friction involves in some cases high stress and strain. Since friction depends strongly on pressure, the friction coefficient has to be measured over a wide range of pressures, while since high stresses have a rejuvenating effect on the polymer, a complex equilibrium is set up between ageing and rejuvenation.

Nonetheless, an overall picture of the relationship between friction and ageing can be drawn from the experiments described in this paper.

At High Pressure

The most straightforward conclusion is that at very high pressure the friction coefficient has a unique value, about 0.45 for PMMA at 30 °C and a sliding speed of 30 μm/s. This value is independent of the thermal history, ageing, and rejuvenation of the polymer. The friction at high pressure induces massive shear yielding at the interface between the material and the sliding tip. It seems to erase any memory of the material history and generate a reproducible surface state.

It will be a challenge to investigate the nature of this erased surface. One can imagine that the molecules in a very thin upper layer are perhaps oriented by the high pressure sliding.

At Low Pressure

At low pressure, close to zero, the situation becomes more complicated and depends on the initial friction coefficient.

- Physical ageing tends to decrease the friction coefficient of a polymer at low pressure. A newly created surface has a friction of about 0.8–1 and a very old surface a friction of approximately 0.25.
- If the initial friction lies above the boundary line calculated by FEM shown in Figure 9, then the tangential stress component produces a high stress at the surface even at low pressure, creating a small volume of sheared and yielded material at the interface between the polymer and the sliding tip. Hence, the surface is immediately sub-

Figure 11. Comparison of the friction properties at 30 °C of newly created surfaces having an equilibrium structural state at 110 °C or 100 °C. The structural state controls the friction at low pressure.
jected to molecular structural changes due to shear and pressure which partly or fully mask its ageing history. This seems to be the reason why the effect of 3 months ageing at 30 °C is not detectable: as the initial friction still lies above the critical boundary even after 3 months, it erases the ageing history.

If the initial friction lies below 0.3, at low pressure the interface between the polymer and the moving tip is not yielded and the effect of ageing on the friction coefficient remains visible as there is no erasing mechanism.

- The ageing time necessary to decrease the friction from more than 0.8 (new surface) to less than 0.3 (below the value producing yielding at the interface) is fairly long. If the friction lies somewhere between 0.3 and 0.8, the results are quite difficult to interpret as ageing and shearing (erasing of the history) compete. FEM simulations indicate that the contact pressure needed to yield the surface increases up to 1.2 as the friction decreases.

- At low pressure, the challenge will be to determine why ageing decreases the friction coefficient. Since ageing leads to a more dense molecular structure, this could provide an explanation.

At Intermediate Pressure

- If the surface is new with a friction well above 0.5, the combined pressure and friction forces produce yielding and modify the molecular state of the surface, giving a friction coefficient strongly dependent on pressure and changing steadily from 0.8 to 0.45 as the latter increases from zero to the yield stress. The molecular state of the surface seems to be controlled by the pressure during sliding.

- If the surface is sufficiently old with a friction well below 0.5, no yielding occurs so long as the pressure remains below the yield stress and hence the friction coefficient is roughly constant at any pressure up to the yield stress. Close to the yield point, the friction increases from its low value to up to 0.45, the stable value for any surface after friction at high pressure.

- In the intermediate pressure range, the challenge will be to determine how the pressure controls the friction.

CONCLUSIONS

Polymer friction displays a strong dependence on temperature, pressure, and sliding speed. The origin of this dependence is often attributed to surface thermodynamics while neglecting the influence of contact mechanics at the local level. Flow line models previously developed have now been used to determine the local shear stress and hence the local friction coefficient. Scratching and sliding experiments were performed on PMMA in the glassy state over a wide range of contact pressures. The friction coefficient was found to vary from less than 0.25 to more than 0.8 and to decrease with physical ageing of the polymer. Whatever the ageing, for a high contact pressure and fully plastic contact, the true friction coefficient tended to a unique value. Physical rejuvenation of the polymer surface seems to offer a plausible explanation for the overall behavior during friction tests if yielding takes place at the contact interface. There is a particular value of the friction coefficient (≈0.5) above which shear yielding takes place even at low contact pressure and below which no shear yielding occurs if the pressure is also below the yield stress. On the basis of these considerations, it was possible to qualitatively rationalize the complex experimental properties of the friction of PMMA.

NOMENCLATURE

- $\mu_{\text{app}}$: apparent friction coefficient
- $\mu$: true friction coefficient
- $\mu_{\text{plough}}$: plowing friction coefficient
- $F_t$: tangential load
- $F_n$: normal load
- $\tau_{\text{int}}$: interfacial shear stress
- $\tau_{\text{adh}}$: adhesive shear stress
- $p$: contact pressure
- $\sigma_{\text{yield}}$: yield stress
- $S_r$: real normal contact area
- $S_t$: tangential contact area
- $ds$: contact surface element
- $\tan \delta$: loss factor
- $E$: Young's modulus
- $i_{\text{vp}}$: generalized viscoplastic strain rate
- $\dot{e}_{\text{vp}}$: generalized viscoplastic strain
- $K$: consistency
Four typical types of behavior may be distinguished, from bottom to top: elastic contact with no groove, elastic contact with a viscoelastic groove, elastoplastic contact and plastic contact (scratching). Typical values of the normalized contact pressure, which is the ratio of the mean contact pressure to the yield stress, are indicated.

The true contact area is the sum of a front area (half disc of radius \( a_f \)) and a rear area (part of the rear half disc).\(^{32}\) The difficulty is to account for this rear contact to relate the true and lowing frictions to the measured apparent friction. In previous work,\(^{35}\) three types of flow line were tested in a new analytical simulation model designed to determine the apparent friction coefficients of conical and spherical tips scratching a surface. The input data required by the simulation are the true contact area, the true friction coefficient, and a model of the pressure acting on the contact surface. The type of pressure distribution used in the model does not significantly modify the results of the simulation.

The elementary local normal and tangential forces due to material flow acting on a contact surface element \( ds \) of the tip are:

\[
\vec{N} = -pds\vec{n} \quad (A1)
\]

\[
\vec{T} = \tau ds\vec{t} \quad (A2)
\]

where \( \vec{n} \) and \( \vec{t} \) are unit vectors normal and tangential to the flow line vectors and \( p \) and \( \tau \) the local normal pressure and shear stress. The mean macroscopic values of the forces may be defined by:

\[
\vec{F}_n = F_n\vec{z} = [(Ap + B\tau)S_n]\vec{z} \quad (A3)
\]

\[
\vec{F}_t = F_t\vec{x} = [(Cp + D\tau)S_n]\vec{x} \quad (A4)
\]

with

\[
A = \frac{1}{S_n} \int \vec{n}\vec{z}ds \quad B = -\frac{1}{S_n} \int \vec{t}\vec{z}ds \quad (A5)
\]

\[
C = \frac{1}{S_n} \int \vec{n}\vec{x}ds \quad D = \frac{1}{S_n} \int \vec{t}\vec{x}ds
\]

where \( \vec{x} \) and \( \vec{z} \) are the unit scratching and indentation axes and \( S_n \) is the normal projected contact area. The true friction \( \mu \) is defined as \( \mu = \tau/p \) and therefore:

\[
\frac{F_t}{F_n} = \mu_{\text{app}} = \frac{C + D\mu}{A + B\mu} \quad (A6)
\]

As there is competition in this apparent friction between an adhesive term and a lowing term,\(^{34} (8) \) may be written as:

\[
\mu_{\text{app}} = \mu + \mu_{\text{ploughing}} \quad \text{where} \quad \mu_{\text{ploughing}} = \frac{-B\mu^2 + \mu(D - A) + C}{A + B\mu} \quad (A7)
\]

Solution of this equation relating the true and apparent frictions requires calculation of the four integrals \( A, B, C, \) and \( D, \) which are the elementary action integrals of the local pressure and shear, together with a knowledge of the rear angle \( \omega, \) the real contact area and the geometry of the tip. \( A, B, C, \) and \( D \) take into account the macroscopic contact shape. In the case of frictionless scratching of plastic materials, the apparent friction is equal to the ratio of \( C \) to \( A \) and the results agree with the well-

Journal of Polymer Science: Part B: Polymer Physics
DOI 10.1002/polb
known analytical solutions for a conical$^{35}$ or spherical tip.$^{36}$ Conversely, if the apparent friction coefficient and shape of the tip are known from experimental data, the true friction may be calculated from eq. A6:

$$\mu = \frac{A \mu_{app} - C}{D - B \mu_{app}}$$ (A8)

This equation was used to estimate the true friction coefficient and the interfacial shear stress, defined as the product of the true friction and the mean contact pressure.

**REFERENCES AND NOTES**