Viscoelastic and elastic–plastic behaviors of amorphous polymeric surfaces during scratch

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In this study, we propose an analysis of the residual groove after contact between a spherical indenter and an amorphous polymeric surface (polymethylmethacrylate, PMMA) in scratch experiments. The geometrical shape of the residual groove was mathematically described using an exponential decay law. Finite element modeling (FEM) of scratch tests was compared to the corresponding experimental results. Assuming a two-segment simplified constitutive law with linear elastic behavior followed by linear strain hardening, the friction at the interface between the indenter and the material was modeled with a Coulomb’s friction coefficient varying from 0 to 0.4, for computed ratios a/R between 0.1 and 0.4. The FEM results for elastic–plastic contact indicate that the shape of the residual groove is directly related to the plastic strain field in the deformation beneath the indenter during scratching. It is shown that the dimensions of the plastically deformed volume and the plastic strain gradient both depend on the ratio a/R and also on the friction coefficient.

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1. Introduction

The scratch resistance of polymer surfaces is becoming critical for the increasing use of these materials in new industrial applications. Due to their low cost, ease of manufacture and processing and low weight, polymers are more and more attractive for an increasing number of industries. However, their lifetime is often reduced by poor surface resistance, for example, to wear and marring [1–3]. In the case of amorphous polymeric surfaces, abrasion causes loss of optical performance. The scratch test with a diamond indenter is commonly employed as a tribological method providing a simplified model of the complex abrasion process, in order to determine the dynamic surface mechanical properties. The scratch resistance of amorphous polymeric surfaces has been the subject of numerous studies [4,5]. Determination of the ductile and brittle behavior during scratching allows the definition of specific parameters, measured with conventional depth-sensing instruments, such as the contact pressure, height of pile-up, or residual depth of the groove [6,7]. As compared to metals or ceramics, polymers exhibit large elastic and viscoelastic contributions to the imposed deformation during both indentation and scratch tests. Furthermore, polymeric materials present a more complex behavior and a finer analysis is required to elucidate the influence on the scratch resistance of not only mechanical but also surface physical properties, such as the true friction coefficient.

A specific experimental scratch device has recently been developed [8] to improve the determination of accurate parameters directly related to the viscoelastic and viscoplastic scratch behavior of amorphous polymers like polymethylmethacrylate (PMMA). Thus, in situ observations during depth-sensing measurements allow study of the viscoelastic recovery of residual imprints during the unloading phase of indentation and likewise at the rear of the contact area in a scratch test (Fig. 1). Moreover, the contact area may be easily distinguished from the residual groove. The main advantage of such a device is that it enables one to record the residual groove profile for a minimum period of time (up to 1 ms) and to observe the evolution of the contact geometry and the morphology of the groove as a function of the scratching parameters, for example the ratio a/R (where a is the contact radius and R the radius of the tip), the sliding velocity V slid or the true friction coefficient µt, as seen in Fig. 1. During scratching of a polymeric surface, various transitions of the mechanical response are observed with increasing applied normal load, from elastic sliding (Fig. 1, zone I, photographs a and b) to viscoplastic scratching (zone III, photograph h). When the ratio (a/R)t lies between 0.2 and 0.4 (zone II-a, photographs c, d and e), the groove initially left on the surface relaxes, corresponding to

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a viscoelastic–plastic contact. At a ratio \((a/R)V\) of about 0.3, a transition can be observed (zone II-a, photograph e) and the material is subjected to both viscoelastic and plastic strains. At the beginning of this new regime, the groove left on the surface has a horizontal imprint and two lateral sloping faces. Lateral plastic pads along the edges of the residual groove and a frontal push pad begin to form for ratios \((a/R)V\) greater than 0.4 (zone II-b, photographs f and g), while the edges of the groove remain parallel.

Combining finite element modeling (FEM) and experimental observations during scratch tests, the present analysis focuses only on the ductile response observed in the zone II-a. The aim was to determine whether the recovery occurring at the rear of the contact area is related to the viscoelastic properties of PMMA, or only due to unloading of the contact caused by the plastic deformation of the volume just beneath the sliding spherical indenter. Assuming a two-segment simplified constitutive law and Coulomb’s friction at the interface between the indenter and the material surface, experimental results were compared to those deduced by FEM, considering in particular the plastic strain field obtained for different ratios \(a/R\) ranging from 0.1 to 0.4 and for true friction coefficients from 0 to 0.4.

2. Experimental procedures

2.1. Experimental set-up

The experimental device for the scratch test, called the ‘micro-visioscratch’, has been described previously [8]. It consists of a commercial servomechanism bearing a small, temperature-controlled transparent box that contains the sample and the scratching tip. Control of the moving tip and recording of the normal \(F_n\) and tangential \(F_t\) loads, scratching speed \(V_{tip}\) and temperature \(T\) are computer driven. A built-in microscope allows in situ observation and measurement of the groove left on the surface (Fig. 2). Scratching over a wide range of speeds \((1–10^4 \text{ m/s})\) and within a temperature range covering the polymer relaxation peaks \((-70 \text{ to } +120 \text{ C})\) are the main innovative features of the system. The normal load \(F_n\) applied to the moving tip can be selected from 0.05 to 35 N by adjusting the compression of a spring of low stiffness. In the present experiments, performed at 30 °C, the speed of the tip was kept constant at 0.03 mm/s. A cone-shaped diamond tip with a spherical extremity was used, having an apex angle of 60° and a tip radius \(R\) of 200 μm. The PMMA surfaces were prepared by different procedures to obtain different values of the true friction coefficient, which was determined for each preparation from experimental results, according to the analytical reverse method of Lafaye et al. [9].

2.2. Finite element model

Scratch tests for a spherical indenter with different radii \(R\) and different penetration depths were modeled using a three-dimensional (3D) finite element code. All calculations were carried out with the implicit FEM package MSC Marc®. A schematic
An illustration of the FEM model for a ratio $a/R$ of 0.4 is presented in Fig. 3. The domain is modeled as a quarter of a cylinder and to limit the number of elements, is reduced to half by means of a symmetry plane ($AABB'$ plane $y = 0$). In contrast to previous studies applying the FEM approach to scratch tests [10–12], remeshing during simulation runs was not performed in the present work. Thus, we developed a specific finite element mesh defined by three zones having different sizes of bilinear isoparametric 8-noded brick elements, using a linear interpolation function (full integration procedure). The simulations required 18,880 elements and 21,384 nodes. In the contact area, the dimensions of the smallest element were about 0.1 times the estimated contact radius $a_0$ during the indentation and scratching phases. The size of the domain ($L_m = 21 \mu m$ and $r_m = 9.4 \mu m$) was chosen to be sufficiently large that boundary effects did not influence the results. The length of the domain of each developed model have been validated in the case of purely elastic contact both during indentation and scratch tests, using several criterion as the evolution of the apparent friction coefficient, the contact geometry and also the stress field. To obtain different ratios $a/R$ ranging from 0.1 to 0.4, the tip radius $R$ and penetration depth $h$ were selected to give for each value of $a/R$ a constant contact radius of about 1.5 $\mu m$. The distance of an indentation from the edge of the sample (along the $y$-axis) was more than six times the contact radius of the indentation, while the thickness of the sample (along the $z$-axis) was at least 30 times the depth of penetration. As boundary conditions, the $x$, $y$ and $z$ displacements of the nodes on the cylindrical surface ($C'CB'B'$) were defined to be null.

The problem was modeled as quasi-static and as such time-independent, with no influence of the strain rate. In order to reproduce experimental tests, the kinematics may be divided into two distinct phases: (i) a first step, corresponding to indentation (along the $z$-axis) at a given penetration depth and (ii) a second step, corresponding to scratching (along the $x$-axis) at a constant relative velocity ($V_{tip} = 6 \mu m/s$) and the fixed indentation depth. The length of the scratch ($L_R = 6 \mu m$, corresponding to a ratio $L_R/a$ of 4) was chosen so that the normal and tangential loads applied to the indenter reached a steady state. In all simulations performed in this study, the spherical indenter was assumed to be rigid and the rheology of the material elastoplastic. The elastoplasticity was defined according to a bilinear von Mises model using isotropic hardening. Since PMMA has a very low hardening tendency at room temperature and small strain rate, a bilinear model with a constant tangent modulus provided a suitable fit for the material parameters.

In the case of elastic, linear hardening plastic material, the stress–strain relation may be represented by

$$
\sigma = \begin{cases} 
E \epsilon & (\sigma \leq \sigma_y) \\
\sigma_y + E_t (\epsilon - \epsilon_y) & (\sigma > \sigma_y)
\end{cases}
$$

where $E$ is the elastic modulus, $\sigma_y$ the yield stress, $\epsilon_y$ the yield strain satisfying $\epsilon_y = \sigma_y/E$ and $E_t$ the tangent modulus.

![Fig. 3. Model of the spherical indenter and the mesh for simulation of scratching.](image-url)
corresponding to the constant work-hardening slope. The elastic modulus, yield stress and tangent modulus were fixed at $E = 3.5$ GPa, $\sigma_y = 100$ MPa and $E_T = 350$ MPa. This simplified model reproduces very closely the true rheology of PMMA. The contact between the spherical indenter and the surface of the sample was enforced using a penalty function method and a geometric description of slave and master surfaces. Unlike in previous studies [10–12], the interface between the rigid indenter and the deformable surface was not assumed to be frictionless. An isotropic Coulomb model was employed to include frictional effects. Thus, the maximum local shear stress acting at the interface $\tau_c$ may be written as

$$\tau_c = \mu p$$

where $p$ is the local normal pressure and $\mu$ the coefficient of the true friction between the surfaces in contact. The contact model was implemented in the context of a finite sliding formulation, where arbitrary sliding and rotation between the surfaces could occur. At each ratio $a/R$, simulations were performed for different values of the true friction coefficient in the range 0–0.4.

3. Results

3.1. Description of the experimental residual groove

As seen in Fig. 4(a), a scratch on a polymer surface generated by a moving tip can be divided into two different parts: (i) the contact area between the indenter and the surface and (ii) the residual groove. The contact area has a front side and a rear side and its shape changes with the ratio of the contact radius $a$ to the tip radius $R$, for a spherical indenter. Similarly, the geometrical shape of the recovery groove appears to be very sensitive to the ratio $(a/R)_y$, as seen in Figs. 1 and 4(a). An analysis of the optical micrographs of the groove left on the polymer surface allowed us to define the true geometrical shape of both the contact area and the recovery groove at the rear of the contact, as shown in Fig. 4(a) for different ratios $(a/R)_y$ corresponding to the zone II-a of Fig. 1. As shown in Fig. 2(b), we were able to plot for the top and bottom edges the half-width $y$ of the groove as a function of its lifetime $t$. The lifetime was estimated from the ratio of the scratch length $x$ to the sliding velocity $V_{tip}$:

$$t = \frac{x}{V_{tip}}$$

In Fig. 4(b), experimental data (symbols) for the top and bottom edges of the groove are plotted for three ratios $(a/R)_y$. The accuracy of such an analysis is related to the resolution of the optical micrographs, which is about 0.7 $\mu$m in both plane directions. In most scratch tests performed with our experimental set-up, this resolution gives largely sufficient accuracy to propose a mathematical fit of the flame at the rear of the contact.

Hence for each ratio $(a/R)_y$, the experimental data $y = f(t)$ were fitted with an exponential decay function defined by

$$y(t) = y_0 + A_1 \exp\left(-\frac{t - t_0}{c_1}\right)$$

where $A_1$ and $c_1$ are two fitting parameters. The values $y_0$ and $t_0$ correspond to the first point of the flame and have to be manually defined for each set of experimental conditions to take into account the elastic deflection of the surface around the contact area. As seen in Fig. 4(b), the experimental data are well described by the fitting function, with a correlation coefficient of more than 0.99.

To study the influence of the ratio $(a/R)_y$ and the local friction coefficient on the recovery groove, we first considered points for...
In the case of the elastic sliding of an indenter with a spherical tip, the distributions of the stress component and the corresponding strain on the surface of a semi-infinite solid are known and the influence of the true friction coefficient has been clearly demonstrated [15]. On the contrary, in the case of elastic–plastic contact with friction, no analytical equations are yet available. Several authors have used 3D finite element approaches to model by stress and strain simulation the mechanical behavior of bulk materials [10–12] or rigid-coated surfaces [16] in scratch tests. Felder and Bucaille [11] drew topographical equivalent plastic strain maps for conical indenters and frictionless contact on bulk materials assumed to be elastic and perfectly plastic (no strain hardening), showing the influence of the semi-apical angle of the conical indenter. The maximal plastic strain attained near the indenter tip was found to vary from 1.5 to 8.4, as the semi-apical angle of θ decreased from 70.3° to 30°. In scratch tests with a spherical tip on polymers, Bucaille et al. [12] described the influence of the strain hardening for contact with a constant friction coefficient (μ = 0.3). At different ratios a/R in the range 0.15–0.6, the plastic strains involved in the deformation during scratching were completely different, depending on the strain hardening. The plastic strain field during scratching with a spherical tip is in fact very complex to describe, as it depends not only on the ratio a/R, but also on the strain hardening of the material. Recently, modeling the contact between a spherical tip and a polymer surface using elliptical contact pressure and shear stress distributions, Gauthier et al. [14] performed numerical simulations to locate the boundaries between (i) elastic and elastic–plastic contact and (ii) elastic–plastic and fully plastic contact. The curve of the evolution of the normalized pressure (Pn/a0) as a function of the local friction for the second boundary showed that the contact yielding depended on the geometrical contact, i.e. the ratio (a/R)0 as proposed initially by Tabor [13], and on the true friction coefficient. These preliminary results indicate that the mean contact strain during scratching must be defined as a function of these two variables. To a first approximation, the mean contact strain may be considered to be proportional to μ and the recovery groove may be analyzed in terms of the true friction coefficient. There is nevertheless no study describing the influence of the friction coefficient on the plastic strain beneath the indenter and in the residual groove.

The true friction between a moving spherical indenter and an elastic–plastic surface not only increases the maximal value of the equivalent plastic strain, but also modifies the shape of the plastically deformed volume and the corresponding plastic strain gradient in the contact zone and the residual groove. At low values of a/R and μ, the plastic strain does not reach the surface but

$t \geq t_0$ and normalized Eq. (4) as follows:

$$y_{\text{norm}}(t) = \frac{1}{y_0 + A_1} \left[ y_0 + A_1 \exp \left( -\frac{t}{\tau_1} \right) \right]$$

Fig. 5(a) and (b) show the evolution of the normalized half-width $y_{\text{norm}}(t)$ as a function of the ratio $(a/R)_{1/2}$ for a frictionless contact ($\mu = 0.075$) and as a function of the true friction coefficient for a given ratio $(a/R)_{1/2} = 0.29$, respectively. One may first note that the curves $y_{\text{norm}}(t)$ vs. $(a/R)_{1/2}$ at $\mu = 0.075$ and $y_{\text{norm}}(t)$ vs. $\mu$ for a given ratio $(a/R)_{1/2}$ are quite similar. The lateral pad described by the function $y_{\text{norm}}(t)$ is the same for $(a/R)_{1/2} = 0.33$ and $\mu = 0.075$ as for a smaller ratio $(a/R)_{1/2} = 0.28$ and a higher friction coefficient $\mu = 0.29$. With increasing ratio $(a/R)_{1/2}$ or true friction coefficient $\mu$, the bottom and top edges of the recovery groove become more parallel and the characteristic time $\tau_1$ increases. In other words, an increase in $(a/R)_{1/2}$ or $\mu$ slows the recovery of the groove. The evolution of the lateral pad as a function of the ratio $(a/R)_{1/2}$ is not really surprising, since with increasing $(a/R)_{1/2}$ the nature of the contact is known to become more plastic, as seen in Fig. 1. The deformation of the surface imposed by the tip geometry becomes less reversible with time, even for polymer surfaces. In the case of materials exhibiting no strain hardening and frictionless contact, the ratio $(a/R)_{1/2}$ is assumed to be simply proportional to the mean contact strain for both indentation and scratching with a spherical indenter [13]. As for the geometrical parameters classically used to describe the contact, i.e. the rear angle $\omega$ or the ratio of the rear length to the front length of the contact area [14], the half-width of the recovery groove depends mainly on the plastic deformation around the contact, i.e. on the mean contact strain. However, the evolution of the lateral pad along the recovery groove as a function of the true friction coefficient for a given ratio $(a/R)_{1/2}$ is original and very interesting. If we suppose that the slope of the residual groove is related to the mean contact strain imposed during contact, Fig. 5(b) clearly demonstrates that the true friction modifies the mean contact strain beneath the moving tip during scratching with a spherical indenter.

### 3.2. Plastic strain field beneath a spherical indenter

In the case of the elastic sliding of an indenter with a spherical tip, the distributions of the stress component and the corresponding strain on the surface of a semi-infinite solid are known and the...
attains its maximal value at a specific depth of about \( z = a/2 \), where \( a \) is the true contact radius (Fig. 6) [14], in good agreement with Hamilton and Goodman’s results for elastic sliding contact [15]. This point corresponds to the maximal shear stress, where plastic deformation may occur. An elastically deformed volume between the rigid indenter and the plastic gradient can be observed when \( a/R \) is less than 0.2. At the surface beneath the indenter and in the residual groove, the contact appears to be mainly elastic. When \( a/R = 0.2 \), which corresponds to classical scratch tests performed experimentally on metals or polymers, the equivalent plastic strain reaches the surface, but its maximal value is still located at a depth of \( z = a/2 \) (Fig. 6(a)). The contact shape and plastic strain gradient are similar when \( a/R = 0.4 \) for frictionless contact (Fig. 6(b)). The plastically deformed volume extends gradually in each case to a maximal depth of 1.5\( a \), the only difference being in the range of plastic strain, which is higher by a factor of about 2 in the second case.

Conversely, for \( a/R = 0.3 \) and a high value of the friction coefficient (\( \mu = 0.4 \)) (Fig. 7), an important difference can be seen in the contact shape and plastic strain gradient. The increase in friction generates not only an increase in the maximal equivalent plastic strain, but also a displacement of its location. The maximal plastic strain is found at the rear of the indenter directly under the contact and in the residual groove, the contact appears to be mainly elastic. When \( a/R = 0.2 \), which corresponds to classical scratch tests performed experimentally on metals or polymers, the equivalent plastic strain reaches the surface, but its maximal value is still located at a depth of \( z = a/2 \) (Fig. 6(a)). The contact shape and plastic strain gradient are similar when \( a/R = 0.4 \) for frictionless contact (Fig. 6(b)). The plastically deformed volume extends gradually in each case to a maximal depth of 1.5\( a \), the only difference being in the range of plastic strain, which is higher by a factor of about 2 in the second case.

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In Fig. 8(a), the evolution of the maximal equivalent plastic strain \( e_{\text{eq max}} \) is shown as a function of the true friction coefficient \( \mu \), for different values of \( a/R \). At low values of \( a/R \) (less than 0.15) no significant influence of the friction coefficient can be observed for \( \mu \leq 0.3 \). However, for \( \mu = 0.4 \) and \( a/R = 0.15 \), the maximal equivalent plastic strain increases by a factor of 2.7, from \( e_{\text{eq max}} = 0.048 \) for \( \mu = 0 \) to \( e_{\text{eq max}} = 0.13 \) for \( \mu = 0.4 \). At higher values of \( a/R (\geq 0.2) \), the maximal equivalent plastic strain becomes even more sensitive to the friction coefficient, for \( \mu > 0.2 \) at \( a/R = 0.2 \) and for \( \mu > 0.1 \) at \( a/R = 0.4 \). At an imposed ratio \( a/R \) of 0.4, the maximal equivalent plastic strain increases from \( e_{\text{eq max}} = 0.18 \) for \( \mu = 0 \) to \( e_{\text{eq max}} = 0.794 \) for \( \mu = 0.4 \). These original results are not obvious because the values of \( e_{\text{eq max}} \) calculated for different ratios \( a/R \) and \( \mu = 0.3 \) are in good agreement with those reported previously by Bucaille et al. [12]. The influence of the true friction coefficient on the maximal equivalent plastic strain is further demonstrated in Fig. 8(b), which depicts the evolution of \( e_{\text{eq max}} \) as a function of the ratio \( a/R \) for contacts with no friction and friction up to \( \mu = 0.4 \).

The curves in Fig. 8(a) and (b) correspond to mathematical descriptions of the evolution of the maximal plastic strain as a function of the friction coefficient and the imposed ratio \( a/R \), respectively. To a first approximation, the maximal plastic strain \( e_{\text{eq max}} \) on equivalent plastic strain maps (Figs. 6 and 7) can be expressed as a function of the true friction coefficient for a given ratio \( a/R \), using an exponential growth law function (Fig. 8(a)):

\[
e_{\text{eq max}} = e_{02} + A_2 \exp \left( \frac{\mu}{t_2} \right)
\]

where the fitting parameters \( e_{02}, A_2 \) and \( t_2 \) depend only on \( a/R \). The mathematical functions related to each parameter are more or less complex. Similarly, the maximal plastic strain varies with the imposed ratio \( a/R \) for a given friction coefficient as a power law function defined by the following relationship (Fig. 8(b))

\[
e_{\text{eq max}} = e_{03} + A_3 \left( \frac{a}{R} \right)^m
\]

where \( e_{03}, A_3 \) and \( m \) are fitting parameters and depend only on the friction coefficient. The parameter \( e_{03} \) and the exponent \( m \) may be considered to be constant, with \( e_{03} \) close to zero and \( m \) about 1.5. \( A_3 \) increases from 0.65 to 3.35 as \( \mu \) varies from 0 to 0.4 and can be expressed as a function of the friction coefficient using an exponential growth law function. In this way, using Fig. 6 and Eqs. (6) and (7), we were able to estimate for PMMA the maximal equivalent plastic strain involved during scratch tests performed with the experimental setup described in section 2.1, as a function of the scratching conditions \( (a/R) \) and \( \mu \). The above Eqs. (6) and (7) will be used in the next section to analyze experimental data.
One should bear in mind that these estimations imply that the scratching contact strain is a complex function defined by two variables, (i) the geometrical contact strain or ratio $a/R$ and (ii) the true friction coefficient $\mu$, as proposed recently by Gauthier et al. [14]. On the other hand, the FEM results presented here suggest that the two components $a/R$ and $\mu$ are not really decoupled and hence for a given ratio $a/R$, the scratching contact strain may not be considered to be simply proportional to the true friction coefficient. Nevertheless, even if no complete solution can be proposed for the scratching contact strain, we may conclude that the friction and the ratio $a/R$ have a comparable effect on the level of equivalent plastic strain. Thus, a scratch experiment with a maximal equivalent plastic strain of $\varepsilon_{\text{max}}^{\text{eq}} = 0.2$ can be obtained under various scratching conditions for a given material, (i) for $\mu = 0$ at $a/R = 0.4$, (ii) for $\mu = 0.2$ at $a/R = 0.3$ and (iii) for $\mu = 0.4$ at $a/R = 0.2$.

Finally, as the ratio $a/R$ and the friction coefficient increase, similar effects are observed on the geometrical shape of the residual flame, with a diminution of the elastic recovery at the bottom of the groove and a tendency for the lateral edges to become more parallel, as seen in Fig. 5. In the same way, with increasing ratio $a/R$ and/or friction coefficient, the plastic strain gradient beneath the indenter and consequently in the residual groove is modified, with an important variation in the maximal plastic strain. The lifetime of the residual groove left on the surface after a scratch experiment appears to be governed by the plastic strain imposed during contact between the moving tip and the surface. The particular geometrical shape of the residual flame, mathematically described by Eq. (5), seems to be related mainly to the elastic unloading of a plastically deformed volume, whose size and plastic strain gradient depend on the ratio $(a/R)_V$ and on the friction coefficient.

4. Discussion

In scratch tests on polymeric surfaces, the geometrical shape of the deformed surface in the rear part of the contact and behind the moving tip in the residual groove appears to be complex, even in the case of a spherical indenter. As shown in Fig. 9 on the 3D topographical representations of the deformed surface computed by FEM, two main phenomena can be distinguished: (i) an elastic deflection of the surface around the contact, especially for a low ratio $a/R$ and a low friction coefficient for a given ratio $a/R$ (Fig. 9(a)) and (ii) the formation of lateral plastic pads along the edges of the residual groove. This latter phenomenon appears to be more pronounced for high values of $a/R$ and a high friction coefficient for a given ratio $a/R$ (Fig. 9(b)). It is interesting to note...
that such features predicted by the finite element approach are often observed experimentally, as seen in Fig. 1. At a given value of the friction coefficient, the elastic deformation around the contact is associated with the regimes in zones I and II-a of Fig. 1, while the formation of lateral pile-up pads corresponds to the regimes in zones II-b and III. In Fig. 9, only the rear part of the contact has been represented for the two scratching conditions and the location \( x/a_0 = 0 \) corresponds to the position of the spherical indenter \( (h/h_0 = 1) \) during the scratching process.

The main difference between the two 3D topographical representations in Fig. 9 concerns the formation of the lateral plastic pads. These pads begin to form at a specific distance from the contact of about 1.5\( a_0 \) for frictionless contact (Fig. 9(a)), which corresponds more or less to the experimental scratch test shown in photograph (e) of Fig. 1 (zone II-a). However, as the friction coefficient increases for the same ratio \( a/R = 0.3 \), one observes a transition of the mechanical response of the surface (Fig. 9(b)). The edges of the residual groove remain more parallel and the lateral plastic pads appear clearly, directly in the contact area and in the residual groove behind the moving tip. The height and thickness of the plastic pads along the groove edges increase with distance from the contact. Under these scratching conditions, the frontal push pad (not shown here) and the lateral plastic pads begin to merge to form a continuous cord. This case simulated by FEM corresponds to the experimental scratch test shown in photograph (f) of Fig. 1 (zone II-b). As depicted in Fig. 7, the variation of the geometrical shape of both the contact area and the residual groove with increasing friction coefficient is directly related to the modification of the plastic strain gradient beneath the indenter and hence in the residual groove, especially in the near-surface region. The top and side views in Fig. 7 show that the yielded volume and the corresponding plastic strain gradient become, respectively, smaller and greater as the friction coefficient varies from \( \mu = 0 \) to 0.4. The good correlation between the transitions observed experimentally with increasing ratio \( a/R \) (Fig. 1) and those computed by simulation with increasing friction coefficient (Fig. 9) supports the hypothesis that the friction and the ratio \( a/R \) have a similar effect on the plastic strain gradient, as suggested in Fig. 8.

Fig. 10 presents the intersection between a horizontal plane and the deformed surfaces for the scratching conditions described in Fig. 9. The horizontal plane was fixed at a constant depth \( z \) corresponding to the penetration depth at the edge of the contact under loading at \( x/a_0 = 0 \) and the FEM results were analyzed using the same procedure as for experimental data (see Section 3.1). The normalized lateral half-width \( y/a_0 \) (where \( a_0 \) is the lateral contact radius at \( x/a_0 = 0 \)) was plotted as a function of the scratching length \( x \) from the contact (where \( x \) is proportional to the lifetime of the groove according to Eq. (3). At the two values of the friction coefficient, the normalized half-width \( y/a_0 \) decreases progressively with the scratching length \( x \), as observed experimentally in Fig. 5. Moreover, with increasing friction coefficient, the edges of the recovery groove computed by simulation become more parallel, as already seen for the analysis of experimental scratch tests, with increasing ratio \( a/R \) (Fig. 5(a)) or friction coefficient (Fig. 7(b)). The curves of Fig. 10(a) correspond to the mathematical description of the normalized half-width as a function of the scratching distance \( x \) using Eq. (4) with the fitting parameter \( t_0 \) set to zero. There is a good correlation between the points calculated by FEM (symbols) and the fit using an exponential decay law. In both experimental and simulated scratch tests, the measured scratching length \( x \) corresponds to approximately twice the contact radius \( a_0 \), with \( a_0 = 60 \) and 1.5\( \mu m \), respectively, for the experimental and simulated data. Fig. 10(a) suggests that the characteristic geometrical shape of the residual flame observed in scratch experiments performed at ratios \( a/R \) of less than 0.4 (zones I and II-a in Fig. 1) is not due to the viscoelastic properties of the material, as initially supposed by Gauthier et al. [14]. The different experimental tendencies are well reproduced by our finite element approach, assuming only elastic–plastic behavior for all the simulated scratching conditions.

In Fig. 10(b), the evolution of the relaxation parameter \( A_1/(y_0 + A_1) \), deduced by fitting of the experimental residual groove using Eq. (5), is represented as a function of the maximal plastic strain \( \varepsilon_{pl} \) varying from 0.22 to 0.41 and friction coefficients ranging from 0.04 to 0.31. The reduced parameter \( A_1/(y_0 + A_1) \) is for us a representative constant parameter describing the ability of the material to relax with time at the rear of the contact after passage of the scratching tip. At high values of the relaxation parameter \( A_1/(y_0 + A_1) \), the groove left on the surface recovers rapidly and after a scratching length exceeding three times the contact radius, there is no residual imprint (photograph c in Fig. 1). At low values of \( A_1/(y_0 + A_1) \), the recovery is slow. The maximal plastic strain imposed in the contact was estimated using Eqs. (6) and (7), describing the evolution of \( \varepsilon_{pl} \), computed by FEM, as a function of the friction coefficient and the ratio \( a/R \), for all scratching conditions tested experimentally (0.22 ≤...
A strong correlation is observed between the relaxation $A_1/(y_0+A_1)$ measured in scratch tests (square open symbols) and the calculated maximal plastic strain $e_{\text{max}}^{eq}$. The relaxation parameter decreases rapidly with the estimated values of the maximal plastic strain, to reach an asymptotic level of about 0.1 for $e_{\text{max}}^{eq}$ greater than 0.2. Similarly, the relaxation parameters deduced by analysis of the simulated $y/a_i$ vs. $x$ curves of Fig. 10(a) have been added to Fig. 10(b) (black circles). At the higher value of the friction coefficient ($\mu = 0.4$), corresponding to a maximal plastic strain of about 0.45, the relaxation parameter seems to be overestimated by simulation. However, for a contact with low friction, the relaxation parameter obtained by FEM is in good agreement with the experimental values.

A complete FEM analysis of the residual groove left on the surface for all simulated scratching conditions is currently in progress with the introduction of a more realistic constitutive law to describe the plastic behavior of amorphous polymeric surfaces and also to study the influence of the yielding criterion of the material beneath the indenter and also on the strain-hardening rule. Even if the elastic linear hardening behavior introduced in FEM may appear to be obvious to reproduce with sufficient accuracy the stress-strain curve of polymeric surfaces, especially at high level of plastic strain, similar FE simulations have been performed using G’sell and Jonas law [17]. Such a constitutive law is known to exhibit an important strain hardening at large strain. Similar tendencies about the evolution of the contact geometry and the shape of the residual groove can be observed as a function of the mean geometrical strain $(a/R)_y$ and the local friction coefficient $\mu$.

In addition, complementary experimental scratch tests have been performed at room temperature to study the influence of the sliding velocity, varying between 0.003 and 0.15 mm/s for a constant geometrical strain $(a/R)_y$ of about 0.3 and a low friction coefficient less than 0.07 (Fig. 11). Using Eq. (5), Fig. 11(a) shows the evolution of the normalized half-width $y_{\text{norm}}(t)$ as a function of the sliding velocity. It is not really obvious to observe such a difference between the curves, because the characteristic time $t_1$ decreases rapidly with the increase of the sliding velocity, with $t_1 = 10.6$ and 0.21 s, respectively, for $V_{\text{tip}} = 0.003$ and 0.15 mm/s. In order to take into account the influence of the sliding velocity, the previous Eq. (5) has been modified, as follows:

$$y_{\text{norm}} \left( \frac{t}{V} \right) = \frac{1}{y_0 + A_1} \left[ y_0 + A_1 \exp \left( - \frac{t}{t_1} \frac{V_0}{V} \right) \right]$$

where $V_0$ is the maximal value of the experimental sliding velocity ($V_0 = 0.15$ mm/s).

As shown in Fig. 11(b) for $(a/R)_y = 0.3$ and $\mu \leq 0.07$, the influence of the sliding velocity on the geometrical shape of the residual groove can be suppressed by using Eq. (8). Fig. 11(b) indicates clearly that the sliding velocity does not control the geometrical shape of the residual groove. This last experimental result is in good agreement with the basic assumption classically used for amorphous polymeric materials for temperature less than the glass temperature. For such materials, the stress–strain curve $\sigma(\varepsilon, T)$ describing the evolution of the stress $\sigma$ as a function of the total strain $\varepsilon$, the strain rate $\dot{\varepsilon}$ and the temperature $T$ can be divided into two distinct functions $\sigma(\varepsilon, T)$ and $f(\varepsilon, T)$. This assumption justifies the choice of the rheology supposing only elastoplastic behavior in first approximation, in order to describe the geometrical shape of the residual groove left on a polymeric surface during scratch experiments with spherical indenter.

5. Conclusion

The present study demonstrates that the main parameter controlling a scratch experiment is the mean contact strain, which is shown to be dependent on the ratio $(a/R)_y$ but also on the friction coefficient. The latter is seen to clearly modify the plastic strain gradient in the material beneath the indenter and also on the surface in the contact region. As the friction coefficient increases, the maximal plastic strain tends to be located directly in the surface region just behind the indenter. Moreover, the friction coefficient $\mu$ and the ratio $(a/R)_y$ play a comparable role. The same level of maximal plastic strain can be obtained for various scratching conditions (values of $(a/R)_y$) and the friction coefficient and consequently to the mean contact strain imposed during scratching. All transitions between (i) elastic sliding and plastic–elastic contact or (ii) elastic–plastic contact and plastic scratching can be described using the finite element approach. The formation of lateral plastic pads along the edges of the residual groove is well reproduced and mainly due to the elastic unloading of the plastically deformed volume. The size of the lateral plastic pads appears to be sensitive to the geometrical shape and dimensions of the plastically deformed volume.
References